

Spectroscopic Amplitudes for One–Nucleon Transfer Between 1p0f–Shell Nuclei

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Abstract. Spectroscopic amplitudes are calculated for one–nucleon transfer between low–lying, normal–parity states of nuclei in the lower part of the 1p0f–shell. Calculations are performed using shell–model wave functions obtained from the diagonalization procedure of a nuclear Hamiltonian in the space given by the complete set of states generated from the $1p_{3/2}$, $1p_{1/2}$, $0f_{7/2}$ and $0f_{5/2}$ orbits. The Hamiltonian contains one and two body interactions derived recently by Richter *et al*. Sum rules for one–nucleon pick–up and stripping reactions are given. The selectivity in excitation of the final states induced by one–nucleon pick–up or stripping is discussed.

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1. Introduction

Over the last thirty years spectroscopic amplitudes (SA's) have been widely employed in many semi-microscopic studies on nuclear structure and transfer reaction mechanisms. These investigations have been performed in the framework of the Distorted Wave Born Approximation (DWBA), because of its advantage to relate in a simple way the kinematic and spectroscopic conditions of transfer reactions (Austern 1970, Satchler 1970, Glendenning 1983).

On the other hand noticeable progress has been attained in the developement of fully microscopic methods to describe nuclear structure and reactions (Wildermuth and Tang 1977, Hofmann 1987) without the necessity to use spectroscopic amplitudes at all. However, in spite of this fact, there are limitations in applying fully microscopic methods to systems with a small number of nucleons. Therefore, SA's are still indispensable in nuclear structure and reaction investigations.

In the literature many papers can be found which consider SA's for nuclei from a wide mass range. The most complete and consistent data exist for 0p-shell nuclei (Cohen and Kurath 1967, Cohen and Kurath 1970, Kurath and Millener 1975, Kurath 1973, Kwaśniewicz and Jarczyk 1985, Kwaśniewicz *et al* 1985). Also a noticeable collection of multinucleon SA's is given for 1s0d-shell nuclei (Conze and Monakos 1979, Glaudemans *et al* 1964, McGrory and Wildenthal 1973, Inoue *et al* 1966, Hecht and Braunschweig 1975, Draayer 1975). On the other hand, for heavier nuclei, the existing data are neither complete nor consistent. In many cases calculations are limited to selected nuclei with wave functions created by imposing various limitations on the model space and by employing various interactions (McGrory 1970, Meuders *et al* 1976, Koops and Glaudemans 1977, Van Hees and Glaudemans 1979, Kutschera *et al* 1978, Poves and Zuker 1981, Muto *et al* 1978, Cole 1985, McCullen *et al* 1964). The necessity to employ various truncated model spaces is dictated by the diagonalization procedure of matrices with enormously large dimensions.

Recently new two-body interactions for 1p0f-shell nuclei have been derived by Richter *et al* 1991. With these interactions the shell-model calculations have been carried out in the full shell-model space generated from the $1p_{3/2}$, $1p_{1/2}$, $0f_{7/2}$ and $0f_{5/2}$ orbits. A satisfactory description of the experimental binding energies, energy spectra, magnetic dipole and electric quadrupole moments of nuclei in the lower part of the 1p0f-shell is obtained. This implies to use the wave functions yielded by the new interactions in calculations of other observable quantities.

The aim of this work is i) to create a consistent set of one-nucleon SA's for low-lying normal parity states of the 1p0f-shell nuclei (in the mass range $41 \leq A \leq 47$) by employing the wave functions obtained with the interaction of Richter *et al* 1991, ii) to show simple examples of applications of these data in predicting the intensity in

population of states of nuclei which are excited in transfer reactions. The restriction to nuclei from the lower part of 1p0 f-shell is imposed by long CPU-time and enormous disc quota required on the computer (CONVEX C3) to solve the eigenvalue problem for heavier nuclei.

The organisation of the paper is as follows. In section 2 a method of calculation for one nucleon SA's for 1p0 f-shell nuclei is given. In section 3 the sum rules for calculated SA's are formulated and selected examples of their application to predict the population of the final states produced in one nucleon pick-up and stripping reactions are presented. The results are summarized in section 4.

2. Formalism

2.1. Spectroscopic amplitudes

The definition of SA's can be found in many papers. According to the notations of Towner 1977 and Ichimura *et al* 1973 the SA's for decomposition of a nucleus A into the core B and a nucleon can be written as†

$$S_{n\ell j}^{\frac{1}{2}} = \sqrt{A} \langle \Phi_{E_A J_A T_A} | (\Phi_{E_B J_B T_B} \times \phi_{n\ell j\tau}(\mathbf{r}))^{J_A T_A} \rangle, \quad (1)$$

where $\Phi_{E_k J_k T_k}$ is the fully antisymmetric, intrinsic wave function of a nucleus k ($k = A$ or B) labelled by the exitation energy E_k , spin J_k and isospin T_k (third components of angular momenta are suppressed) and $\phi_{n\ell j\tau}(\mathbf{r})$ is the wave function of a nucleon in the state of relative motion with respect to the nucleus B specified by the number of nodes n and angular momentum ℓ . The quantum numbers j and τ define the spin ($\mathbf{j} = \ell + \mathbf{s}$) and charge of a nucleon. The coordinate \mathbf{r} denotes the distance between the center of mass of nucleus B and the separated nucleon. Assuming that the intrinsic nuclear wave functions $\Phi_{E_k J_k T_k}$ are approximated by the internal part of the shell-model wave functions $\Psi_{E_k J_k T_k}$, equation (1) can be transformed to the following expression (Towner 1977)

$$S_{n\ell j}^{\frac{1}{2}} = \sqrt{A} \left(\frac{A}{A - 1} \right)^{\frac{2n+\ell}{2}} \langle \Psi_{E_A J_A T_A} | (\Psi_{E_B J_B T_B} \times \phi_{n\ell j\tau}(\mathbf{r}_c))^{J_A T_A} \rangle, \quad (2)$$

where now the wave function $\phi_{n\ell j\tau}(\mathbf{r}_c)$ depends on the coordinate \mathbf{r}_c of the separated nucleon in the laboratory frame of reference.

In this work the SA's are considered for normal parity states of nuclei belonging to the $(1p0f)^n$ shell-model space generated from the $1p_{3/2}$, $1p_{1/2}$, $0f_{7/2}$ and $0f_{5/2}$ orbits. Thus the basis states can be written

$$|\rho\Gamma\rangle = |(\text{core})^{m_c} (\sigma_1^{m_1^\rho})(\sigma_2^{m_2^\rho})(\sigma_3^{m_3^\rho})(\sigma_4^{m_4^\rho}) \Gamma_1^\rho \Gamma_2^\rho \Gamma_3^\rho \Gamma_4^\rho \Gamma_{12}^\rho \Gamma_{34}^\rho; \Gamma\rangle, \quad (3)$$

† The capital letters A and B denote the nucleus or its mass number depending on the context

where ρ is the running index used to number basis states. The σ_k ($k = 1, 2, 3$ or 4) represent a set of quantum numbers n_k, ℓ_k and j_k to define subshells $1p_{3/2}$, $1p_{1/2}$, $0f_{7/2}$ and $0f_{5/2}$, respectively, and m_k^ρ describes the number of nucleons in the subshell σ_k whereas m_c is the number of nucleons in the entirely filled up core. The Γ_k^ρ gives a set of spin and isospin quantum numbers (and additional quantum numbers) describing a configuration of m_k^ρ nucleons in the subshell σ_k . The symbols $\Gamma_{12}^\rho = \Gamma_1^\rho + \Gamma_2^\rho$ and $\Gamma_{34}^\rho = \Gamma_3^\rho + \Gamma_4^\rho$ denote the intermediate coupling spin and isospin angular momenta quantum numbers. Finally $\Gamma = \Gamma_{12} + \Gamma_{34}$ defines the total spin and isospin of the state $|\rho\Gamma\rangle$. Expanding the wave function $\Psi_{E_k J_k T_k}$ in the basis (3) the SA's of equation (2) for a separation of one nucleon from the orbital σ_k have the form

$$S^{\frac{1}{2}}(\sigma_k) = \sqrt{A} \left(\frac{A}{A-1} \right)^{\frac{2n_k+\ell_k}{2}} \sum C_A(i) C_B(f) S_{if}^{\frac{1}{2}}(\sigma_k), \quad (4)$$

where C_A and C_B are the expansion coefficients of wave functions of nuclei A and B . The overlap integral

$$\begin{aligned} S_{if}^{\frac{1}{2}}(\sigma_k) = & \langle (\text{core})^{m_c} (\sigma_1)^{m_1^i} (\sigma_2)^{m_2^i} (\sigma_3)^{m_3^i} (\sigma_4)^{m_4^i} \Gamma_1^i \Gamma_2^i \Gamma_3^i \Gamma_4^i \Gamma_{12}^i \Gamma_{34}^i \Gamma_A | \\ & \times | [(\text{core})^{m_c} (\sigma_1)^{m_1^f} (\sigma_2)^{m_2^f} (\sigma_3)^{m_3^f} (\sigma_4)^{m_4^f} \Gamma_1^f \Gamma_2^f \Gamma_3^f \Gamma_4^f \Gamma_{12}^f \Gamma_{34}^f \Gamma_B] \times (\sigma_k)]^{\Gamma_{\hat{A}}} \rangle \end{aligned} \quad (5)$$

describes a nucleon transition between basis states $|i\Gamma_A\rangle$ and $|f\Gamma_B\rangle$. The explicit formulae for the $S_{if}(\sigma_k)$ are given in the appendix.

2.2. Sum rules

The sum rules for one-nucleon SA's have first been considered by Macfarlane and French 1960. However, these sum rules have been derived only for particular cases of nuclear states expanded in the basis generated from one to two active orbits. In this work nuclear states are considered in a complete basis generated from four active orbits in the $1p0f$ -shell and the sum rules for SA's calculated in the full $1p0f$ shell-model space have to be considered.

The sum rule for one-nucleon pick-up from the target A leading to states of a nucleus $B = A-1$ is given by

$$\sum_B S_{A \rightarrow B=A-1}(\sigma_k) = \sum_i C_A^2(i) m_k^i = \langle m_k \rangle, \quad (6)$$

where the summation on the left-hand side runs over all states of the nucleus B which give non-zero SA's for one-nucleon pick-up from the orbit σ_k in the nucleus A . The expansion coefficients of the wave function of a nucleus A in the ground state are denoted by C_A . The number of nucleons in the orbit σ_k of the basis state $|i\Gamma_A\rangle$ is given by m_k^i . The sum rule of equation (6) gives an average number $\langle m_k \rangle$ of nucleons in the orbit σ_k which can take part in one-nucleon pick-up from the target nucleus A . If additionally

a summation over all active orbits σ_k in both sides of equation (6) is performed one obtains

$$\sum_{B,\sigma_k} S_{A \rightarrow B=A-1}(\sigma_k) = n, \quad (7)$$

where n is the total number of nucleons in all active orbits in the nucleus A .

The sum rule (7) provides a very simple check for calculations of SA's. Furthermore a total strength (Kurath and Millener 1975, Kwaśniewicz and Jarczyk 1985) is defined, i.e. a quantity which can be useful in predicting the intensity of the population of states of residual nuclei produced by one-nucleon pick-up. This will be discussed in section 3.2.

The sum rule for one-nucleon stripping on a target A leading to states of a nucleus $B = A+1$ is given by

$$\sum_B \left(\frac{\hat{\Gamma}_B}{\hat{\Gamma}_A} \right)^2 S_{A \rightarrow B=A+1}(\sigma_k) = \sum_i C_A^2(i) (N(\sigma_k) - m_k^i), \quad (8)$$

where $\hat{\Gamma}_x = \sqrt{(2J_x + 1)(2T_x + 1)}$ ($x = A$ or B) and $N(\sigma_k) = 2(2j_k + 1)$ with j_k denoting the total spin of a nucleon in the orbit σ_k . In a manner similar to equation (7) we sum over all active orbits σ_k and obtain from equation (8)

$$\sum_{B,\sigma_k} \left(\frac{\hat{\Gamma}_B}{\hat{\Gamma}_A} \right)^2 S_{A \rightarrow B=A+1}(\sigma_k) = \sum_{\sigma_k} N(\sigma_k) - n + 1, \quad (9)$$

where n is the total number of nucleons in active orbits in the final nucleus $B = A + 1$. Similar to equation (7) the generalized sum rule (9) provides an additional check for calculations of SA's for one-nucleon stripping and can be employed in predicting the population of states of final nucleus B produced by one-nucleon stripping on the target A (see discussion in section 3.2.).

The sum rules of equations (6–9) have been obtained with the help of formulae (4) and (A4–A7) by employing the orthonormality conditions for the U-coefficients and for the coefficients of fractional parentage (c.f.p.) $\langle (\sigma_k)^{m_k} \Gamma_k | \} (\sigma_k)^{m_k-1} \Gamma_k \rangle$. In addition, a particle-hole relation for c.f.p. (MacFarlane and French 1960), i.e. the relation

$$\begin{aligned} \langle (\sigma_k)^{m_k} \Gamma_k^f | \} (\sigma_k)^{m_k-1} \Gamma_k^i \rangle &= \frac{(N(\sigma_k) - m_k + 1)(\hat{\Gamma}_k^i)^2}{m_k(\hat{\Gamma}_k^f)^2} \\ &\times \langle (\sigma_k)^{(N(\sigma_k)-m_k+1)} \Gamma_k^i | \} (\sigma_k)^{N(\sigma_k)-m_k} \Gamma_k^f \rangle, \end{aligned} \quad (10)$$

where index $i(f)$ corresponds to the nucleus A ($B = A + 1$), has been utilized to derive the sum rules for stripping. Furthermore the orthonormality and closure relations for the expansion coefficients of the nuclear wave functions have been exploited in deriving the sum rules for pick-up and stripping. The sum rules (6–9) relate to SA's of equation (4) where the $(A/A - 1)^{(2n+\ell)/2}$ factor is ignored.

3. Results and discussion

3.1. Calculations

The SA's for one-nucleon transfer have been calculated for nuclei in the mass range $41 \leq A \leq 47$ according to equation (4) and equations (A4–A7). The eigenfunctions of nuclear states considered have been obtained with the help of the program RITSSCHIL (Zwarts 1985) by diagonalization of a nuclear hamiltonian in the space defined by equation (3). The upper limit of the mass number is constrained because of the long CPU time and enormous disc quota required on the CONVEX C3 computer to calculate the eigenenergies and eigenfunctions. The single particle energies for $1p_{3/2}$, $1p_{1/2}$, $0f_{7/2}$ and $0f_{5/2}$ orbits and two-body matrix elements, i.e. the FPD6 interactions derived by Richter *et al* 1991 have been employed in the present calculations. The procedure for the mass-dependence correction of the two body matrix elements (Richter *et al* 1991) has been taken into consideration. A selected example of the spectroscopic amplitudes for one-nucleon stripping from $^{45}\text{Sc}(7/2^-3/2)$ is shown in table 1. We have also calculated SA's for one-nucleon transfer between excited states of both colliding nuclei. These data can be useful for studying more sophisticated transfer processes, e. g. processes which occur inside stars. The SA's for one-nucleon pick-up and stripping for target nuclei being in the ground and excited states can be obtained upon request from one of the authors (H.H.).

3.2. The intensity of states populated in one-nucleon transfer

Transfer reactions show a remarkable degree of selectivity by occupying only a few states in the residual nuclei with any appreciable strength. This provides an attractive way to relate the dynamic and spectroscopic conditions which are clearly exhibited in the framework of the Distorted Wave Born Approximation (DWBA) for direct nucleon transfer. The transfer amplitude in the simplest case is a product of the dynamic form factor calculated with DWBA in the optical model and the SA (Austern 1970, Satchler 1970, Glendenning 1983). Due to this factorization one can determine the spectroscopic factor (i.e. square of the SA) as a quotient of the measured value of the cross section $d\sigma/d\Omega$ and the theoretical value of the squared DWBA dynamic form factor $\sigma^{\text{DWBA}}(\Theta)$ (Glendenning 1983). The spectroscopic factor determined in this way will be denoted as the experimental spectroscopic factor. It can be compared with the theoretical spectroscopic factor obtained from a shell-model description of the initial and final nuclear states.

In this section we focus on an examination of the spectroscopic conditions which are reflected in the nuclear overlap integrals considered in section 2. Neglecting the dynamic conditions the transition intensity can be deduced from considering the percentage

distribution of the total strength (defined for pick-up by the sum rule of equation (7) and for stripping by the sum rule of equation (9)) among states populated by one-nucleon transfer.

Figure 1 illustrates an example where almost all total strength is deposited in the lowest states of the residual nucleus. In a nucleon pick-up from ^{43}Ca 94% of the total strength is deposited in the six lowest lying states of ^{42}Ca from among which the first $J^\pi T = 0^+1$ and the first 6^+1 states absorb 25 % and 33% of the total strength respectively. A striking example is neutron pick-up from the ^{44}Ca target. In this case the calculation of the SA's gives that 91% of the total strength is deposited in the first $7/2^-3/2$ state of the residual nucleus ^{43}Ca .

Examples where around half of the total strength is deposited in the first few states of the residual nucleus are presented in figures 2 and 3. The neutron pick-up from the ^{45}Sc target gives 50% of the total strength distributed among the first 2^+1 (7%), 6^+1 (8%), 4^+1 (7%), 1^+1 (6%), 7^+1 (22%) states of the final ^{44}Sc nucleus (figure 2).

Theoretical predictions are in excellent agreement with experimental findings (see table 2) obtained from studying the (d,t) reaction on the ^{45}Sc target (Ohnuma and Sourkes 1971). Here we want to mention that the spectroscopic factor for the isobaric analog of the $^{44}\text{Ca}_{\text{g.s.}}$ in ^{44}Sc ($J^\pi T$, $E_{\text{exp}} = 0^+2$, 2.783 MeV) extracted from $^{45}\text{Sc}(^3\text{He},\alpha)^{44}\text{Sc}$ (Rappaport *et al* 1971) and $^{45}\text{Sc}(\text{d},\text{t})^{44}\text{Sc}$ (Ohnuma and Sourkes 1971) is equal to 0.5 and 1.1, respectively, while from our calculations a value of 0.55 is obtained. In the 0^+2 state excited by neutron pick-up from the ^{45}Sc target 11% of the total strength is deposited. In the first excited state of ^{45}Sc ($J^\pi T$, $E_{\text{theor}} = 2^+2$, 4.755 MeV) only 1% and in the next 10 excited $T = 2$ states our shell-model calculations predict altogether less than 1% of the total strength. About a quarter of the total strength is distributed among the seven lowest states of ^{47}Ca produced by the neutron stripping on the ^{46}Ca target (figure 3). A total strength of 23% is deposited in the first $7/2^-7/2$ (7%), the first $3/2^-7/2$ (11%) and the second $1/2^-7/2$ (5%) states of ^{47}Ca .

An example of relatively small concentration of the total strength in the lowest lying states of the residual nucleus is illustrated in figure 4. Namely, considering the neutron stripping on the ^{45}Sc target, only 12.9% of the total strength is distributed among the seven lowest states of the ^{46}Sc nucleus. The experimental transition strength $(2J_f + 1)S/(2J_i + 1)$ for above mentioned states of ^{46}Sc which are induced by the (d,p) (Roy *et al* 1992, Rappaport *et al* 1966, Bing *et al* 1976) and (t,d) (Brussaard and Glaudemans 1977) reactions on the ^{45}Sc target is compared with the theoretical predictions (figure 4 and table 3). Theoretical results of the present work are also compared with the earlier results obtained by employing the truncated shell-model space generated from the $0 f_{7/2}$ orbit only (McCullen *et al* 1964). Because of experimental uncertainties (Roy *et al* 1992) it is difficult to determine which theoretical calculations better describe the experimental data. It seems that both theoretical calculations (figure

4) are within limits of uncertainty in good agreement with the experimental results.

In order to get a better insight into the quality of the present calculations, the theoretical predictions of spectroscopic factors of one-nucleon pick-up and stripping are compared with the experimental findings in tables 2 and 3. As already mentioned, the experimental spectroscopic factors were obtained under the assumption that only one orbital (nlj) contributes to the cross section. This assumption seems to be justified since the magnitudes of the theoretical SA's assigned to this orbital considerably exceeds the ones assigned to the remaining allowed orbitals. Therefore, for instance, in the $^{45}\text{Sc}(\text{d},\text{t})^{44}\text{Sc}$ and $^{45}\text{Sc}(\text{d},\text{p})^{46}\text{Sc}$ reactions almost only the $nlj = 0\ 3\ 7/2$ orbital is populated.

Spectroscopic factors deduced from one-nucleon transfer reactions are as a rule sensitive to the radius of the potential in which the bound-state wave function is generated as well as to the optical model potential describing wave functions in the entrance and exit reaction channel.

During the last decade many nuclei have been studies with electron-induced proton knock-out ($e,e'p$)-reactions (de Witt Huberts 1990). An important advantage is that the spectroscopic factors deduced from these reactions are not sensitive to the shape of the single-particle binding potential and to the optical potential describing the scattering state of the outgoing proton. However, the spectroscopic factors determined from ($e,e'p$)-reactions are surprisingly small compared to results obtained with transfer reactions, sometimes even by 30–40% (de Witt Huberts 1990). Theoretical investigations initiated to understand this effect (Pandharipande 1989, Benhar *et al* 1989, Van Neck *et al* 1991, Wesseling *et al* 1992) indicate that short-range correlations and uncertainties of the ($e,e'p$)-reaction mechanism, in particular due to the final state interaction sector may be responsible for this effect (de Witt Huberts 1990). The lack of reliability of the approximations inherent in these models, in particular for light nuclei and medium-mass systems, still does not seem to allow to make significant conclusions on possible differences of spectroscopic factors deduced from nuclear transfer and ($e,e'p$)-reactions (for a more detailed discussion see de Witt Huberts 1990). Anyway, the ($e,e'p$)-reactions as a spectroscopic tool exhibit more model sensitivity than transfer reactions when studying phenomena of multinucleon systems .

3.3. Fluctuations in spectroscopic factors

Some applications of the sum rules for SA's allow obtaining valuable informations on nuclear structure without performing advanced shell-model calculations. As an illustrative example can serve the case when the summations in the sum rules (7) and (9) reduce to a single term yielding expressions for individual spectroscopic factors. Due to this property fluctuations in spectroscopic factors throughout a shell/subshell can be

demonstrated.

Consider m identical nucleons occupying a single shell σ_k above the ^{40}Ca core. Applying equation (9) to the $(\sigma_k)_{J=j_k}^{2m_k+1} \rightarrow (\sigma_k)_{J=0}^{2m_k}$ transition one obtains

$$(2j_k + 1)S(2m_k + 1 \rightarrow 2m_k) = (2m_k + 1) - 2m_k. \quad (11)$$

Similarly equation (7) yields for the $(\sigma_k)_{J=0}^{2m_k} \rightarrow (\sigma_k)_{J=j_k}^{2m_k-1}$ transition

$$S(2m_k \rightarrow 2m_k - 1) = 2m_k. \quad (12)$$

Thus for the chain of transitions

$$(m_k J) = (00) \leftarrow (1j_k) \leftarrow (20) \leftarrow (3j_k) \dots$$

expressions (11) and (12) combine to give

$$S(m_k \rightarrow m_k - 1) = \begin{cases} 1 - \frac{m_k - 1}{2j_k + 1} & \text{for } m_k \text{ odd,} \\ m_k & \text{for } m_k \text{ even.} \end{cases} \quad (13)$$

With the aid of equations (13) the chain of spectroscopic factors for $(\sigma_k)^{m_k} = (0f_{7/2})^{m_k}$ configurations concerning the ground states in Ca can immediately be calculated (table 4). A comparison of these spectroscopic factors with their equivalents calculated in the full $(1p0f)^m$ shell-model space (table 4) allows to conclude that the structure of the ground states of ^{41}Ca – ^{45}Ca nuclei is dominated by the $(0f_{7/2})^m$ configuration. This is contrary to the structure of the ground states of ^{46}Ca and ^{47}Ca nuclei which differs considerably from a simple $(0f_{7/2})^m$ configuration.

4. Summary

Spectroscopic amplitudes for one-nucleon pick-up and one-nucleon stripping reactions with nuclei from the lower part of 1p0f-shell are calculated with the help of wave functions expanded in the complete basis generated from the $1p_{3/2}$, $1p_{1/2}$, $0f_{7/2}$ and $0f_{5/2}$ orbits. The two-body mass-dependent FPD interactions recently derived by Richter *et al* 1991 have been adapted in the calculations of the wave functions. The sum rules for calculated SA's have been derived and examples of their application in predicting the intensity of states of the final nucleus produced by one-nucleon pick-up and stripping reactions are discussed. The examples indicate that for some reactions nearly 100% of the total strength is concentrated in a few low-lying states of the residual nucleus (see figure 1). However, there are examples where the total strength is distributed over a wider energy range of excited states of the final nucleus.

The presented example indicate that the distribution of the total strength among residual 1p0f-shell nuclei is similar to that for the lighter nuclei (see for example Cohen and Kurath 1967, Cohen and Kurath 1970, Kurath and Millener 1975, Kurath 1973, Kwaśniewicz and Jarczyk 1985, Kwaśniewicz *et al* 1985).

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Appendix A.

In this subsection a method of deriving the formulae for the partial spectroscopic amplitudes $S_{if}^{\frac{1}{2}}(\sigma_k)$ of equation (5) is sketched. This is demonstrated for the simplest case, i.e. when a nucleon is transferred from the last subshell σ_4 . Introducing the diagrammatic notation of Macfarlane and French 1960 the overlap integral $S_{if}^{\frac{1}{2}}(\sigma_4)$ of equation (5) is defined below:

(A1)

In order to single out the groups of nucleons $(\sigma_k)^{m_k}$ ($k=1,2,3$ or 4) on both sides of (A1), wave functions have to be constructed that are antisymmetric in these groups separately. Following the procedure described in Brussaard and Glaudemans 1977 one

obtains

(A2)

Employing the one-nucleon expansion for the $(\sigma_4)^{m_4^i}$ and performing a change of angular momenta coupling order the bra wave function of equation (A2) can be written

(A3)

From equations (A2) and (A3) one immediately obtains

$$S_{if}^{\frac{1}{2}}(\sigma_4) = \left(\frac{m_4^i}{A} \right)^{\frac{1}{2}} \langle (\sigma_4)^{m_4^i} \Gamma_4^i | \} ((\sigma_4)^{m_4^i - 1} \Gamma_4^f \rangle$$

$$\begin{aligned} & \times U(\Gamma_3^i \Gamma_4^f \Gamma_{34}^i \sigma_4; \Gamma_{34}^f \Gamma_4^i) U(\Gamma_{12}^i \Gamma_{34}^f \Gamma_A \sigma_4; \Gamma_B \Gamma_{34}^i) \\ & \times \delta_{\Gamma_1^i \Gamma_1^f} \delta_{\Gamma_2^i \Gamma_2^f} \delta_{\Gamma_{12}^i \Gamma_{12}^f} \delta_{\Gamma_3^i \Gamma_3^f} \delta_{m_1^i m_1^f} \delta_{m_2^i m_2^f} \delta_{m_3^i m_3^f} \delta_{m_4^i - 1 m_4^f}. \end{aligned} \quad (\text{A4})$$

In the same way as above the following expressions are derived

$$\begin{aligned} S_{if}^{\frac{1}{2}}(\sigma_3) = & \left(\frac{m_3^i}{A} \right)^{\frac{1}{2}} (-)^{\Gamma_3^i + \Gamma_3^f - \Gamma_{34}^i - \Gamma_{34}^f - m_4^i} \langle (\sigma_3)^{m_3^i} \Gamma_3^i | \} (\sigma_3)^{m_3^i - 1} \Gamma_3^f \rangle \\ & \times U(\Gamma_4^i \Gamma_3^f \Gamma_{34}^i \sigma_3; \Gamma_{34}^f \Gamma_3^i) U(\Gamma_{12}^i \Gamma_{34}^f \Gamma_A \sigma_3; \Gamma_B \Gamma_{34}^i) \\ & \times \delta_{\Gamma_1^i \Gamma_1^f} \delta_{\Gamma_2^i \Gamma_2^f} \delta_{\Gamma_{12}^i \Gamma_{12}^f} \delta_{\Gamma_4^i \Gamma_4^f} \delta_{m_1^i m_1^f} \delta_{m_2^i m_2^f} \delta_{m_3^i - 1, m_3^f} \delta_{m_4^i m_4^f} \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} S_{if}^{\frac{1}{2}}(\sigma_2) = & \left(\frac{m_2^i}{A} \right)^{\frac{1}{2}} (-)^{m_3^i + m_4^i + \Gamma_{12}^i + \Gamma_{12}^f - \Gamma_A - \Gamma_B} \langle (\sigma_2)^{m_2^i} \Gamma_2^i | \} (\sigma_2)^{m_2^i - 1} \Gamma_2^f \rangle \\ & \times U(\Gamma_1^i \Gamma_2^f \Gamma_{12}^i \sigma_2; \Gamma_{12}^f \Gamma_2^i) U(\Gamma_{34}^i \Gamma_{12}^f \Gamma_A \sigma_2; \Gamma_B \Gamma_{12}^i) \\ & \times \delta_{\Gamma_1^i \Gamma_1^f} \delta_{\Gamma_3^i \Gamma_3^f} \delta_{\Gamma_4^i \Gamma_4^f} \delta_{\Gamma_{34}^i \Gamma_{34}^f} \delta_{m_1^i m_1^f} \delta_{m_2^i - 1, m_2^f} \delta_{m_3^i m_3^f} \delta_{m_4^i m_4^f} \end{aligned} \quad (\text{A6})$$

and

$$\begin{aligned} S_{if}^{\frac{1}{2}}(\sigma_1) = & \left(\frac{m_1^i}{A} \right)^{\frac{1}{2}} (-)^{m_2^i + m_3^i + m_4^i + \Gamma_1^i + \Gamma_1^f - \Gamma_A - \Gamma_B} \langle (\sigma_1)^{m_1^i} \Gamma_1^i | \} (\sigma_1)^{m_1^i - 1} \Gamma_1^f \rangle \\ & \times U(\Gamma_2^i \Gamma_1^f \Gamma_{12}^i \sigma_1; \Gamma_{12}^f \Gamma_1^i) U(\Gamma_{34}^i \Gamma_{12}^f \Gamma_A \sigma_1; \Gamma_B \Gamma_{12}^i) \\ & \times \delta_{\Gamma_3^i \Gamma_3^f} \delta_{\Gamma_4^i \Gamma_4^f} \delta_{\Gamma_{34}^i \Gamma_{34}^f} \delta_{\Gamma_2^i \Gamma_2^f} \delta_{m_1^i - 1, m_1^f} \delta_{m_2^i m_2^f} \delta_{m_3^i m_3^f} \delta_{m_4^i m_4^f} . \end{aligned} \quad (\text{A7})$$

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Tables and table captions

Table 1. Spectroscopic amplitudes for one-nucleon stripping from $^{45}\text{Sc}(7/2^-3/2)$. States of the final nucleus are specified by the spin, parity and isospin in the first column and calculated excitation energy in the second column. The orbitals of the transferred nucleon are given in the next four columns. In order to be in an agreement with the definition of equation (1) all listed spectroscopic amplitudes have to be multiplied by the $(A/(A - 1))^3/2$ factor.

$J^\pi T$	E_{calc} (MeV)	$P_{1/2}$	$P_{3/2}$	$F_{5/2}$	$F_{7/2}$	percentage of strength
1^+2	1.788			0.169	-0.294	0.15
1^+2	2.521			0.210	0.131	< 0.1
2^+2	0.228		0.099	-0.122	0.554	0.7
2^+2	1.679		0.116	-0.022	-0.063	< 0.1
3^+2	0.203	0.042	-0.118	0.061	-0.703	1.6
3^+2	1.278	-0.021	-0.031	0.041	0.226	0.17
4^+2	0.0	0.024	-0.065	0.087	-0.598	1.5
4^+2	0.784	0.036	0.084	-0.004	0.489	1.0
5^+2	0.331		-0.185	-0.048	-0.600	1.9
5^+2	0.894		-0.057	0.053	-0.490	1.2
6^+2	0.019			0.026	-0.866	4.4
7^+2	0.892				0.449	1.4

Table 2. A comparison of the experimental and calculated spectroscopic factors for selected one-nucleon pick-up reactions. States of the final nucleus are specified by the experimental excitation energy in the second column and spin and parity in the third column. Orbitals of the transferred nucleon are given in the fourth column.

final nucleus	E_x (MeV)	J^π	ℓj	S (exp.)	S (theor.)
^{41}Ca	0.0	$7/2^-$	$3\ 7/2$	1.46(i) (1.60 (ii))	1.81
	1.942	$3/2^-$	$1\ 3/2$	0.15(ii)	0.11
^{43}Ca	0.0	$7/2^-$	$3\ 7/2$	3.3(iii) (3.5 (iv))	3.64
	0.373	$5/2^-$	$3\ 5/2$	0.14(iii) (0.27 (iv))	0.03
	0.593	$3/2^-$	$1\ 3/2$	0.09(iii) (0.14 (iv))	0.02
	2.046	$3/2^-$	$1\ 3/2$	0.14(iii) (0.31 (iv))	0.14
^{44}Sc	0.0	2^+	$3\ 7/2$	0.35(v)	0.35
	0.271	6^+	$3\ 7/2$	0.48(v)	0.39
	0.350	4^+	$3\ 7/2$	0.35(v)	0.34
	0.667	1^+	$3\ 7/2$	0.32(v)	0.29
	0.763	(3^+)	$3\ 7/2$	0.20(v)	0.03
	0.968	7^+	$3\ 7/2$	1.29(v)	1.09
	1.052	(5^+)	$3\ 7/2$	0.25(v)	0.14
	2.783	$0^+ T = 2$	$3\ 7/2$	1.1(v) (0.5(vi))	0.55

- (i) The $^{42}\text{Ca}(\text{p},\text{d})^{41}\text{Ca}$ reaction ($E_{\text{p}}=26.5$ MeV) from Brown *et al* 1974
- (ii) The $^{42}\text{Ca}(^3\text{He},\alpha)^{41}\text{Ca}$ reaction ($E_{^3\text{He}}=18$ MeV) from Brown *et al* 1974
- (iii) The $^{44}\text{Ca}(\text{p},\text{d})^{43}\text{Ca}$ reaction ($E_{\text{p}}=26.5$ MeV) from Brown *et al* 1974
- (iv) The $^{44}\text{Ca}(^3\text{He},\alpha)^{43}\text{Ca}$ reaction ($E_{^3\text{He}}=18$ MeV) from Brown *et al* 1974
- (v) The $^{45}\text{Sc}(\text{d},\text{t})^{44}\text{Sc}$ reaction ($E_{\text{p}}=19.5$ MeV) from Ohnuma and Sourkes 1971
- (vi) The $^{45}\text{Sc}(^3\text{He},\alpha)^{44}\text{Sc}$ reaction from Rappaport *et al* 1971

Table 3. A comparison of the experimental and theoretical transfer strengths $(2J_f + 1)S$ for selected one-nucleon stripping reactions. For explanation see caption to table 2.

final nucleus	E_x (MeV)	J^Π	ℓj	$(2J_f + 1)S(\text{exp.})$	$(2J_f + 1)S(\text{theor.})$
^{43}Ca	0.0	$7/2^-$	$3\ 7/2$	5.5(i)	6.0
	0.593	$3/2^-$	$1\ 3/2$	0.21(i)	0.05
^{45}Ca	0.0	$7/2^-$	$3\ 7/2$	3.40(ii)	4.03
	1.435	$3/2^-$	$1\ 3/2$	0.47(ii)	0.13
	1.90	$3/2^-$	$1\ 3/2$	2.60(ii)	3.39
	2.249	$1/2^-$	$1\ 1/2$	0.36(ii)	0.24
^{46}Sc	0.0	4^+	$3\ 7/2$	2.82(iii) (4.64 (iv))	3.22
	0.052	6^+	$3\ 7/2$	6.90(iii) (10.64 (iv))	9.75
	0.228	3^+	$3\ 7/2$	2.86(iii) (4.96 (iv))	3.46
	0.281	5^+	$1\ 3/2, 3\ 7/2$	0.44, 4.29(iii) (0.8 (iv))	0.38, 3.96
	0.444	2^+	$3\ 7/2$	1.60(iii) (2.48 (iv))	1.53
	0.774	5^+	$3\ 7/2$	2.78(iii) (4.88 (iv))	2.64
	0.835	4^+	$3\ 7/2$	1.68(iii) (2.56 (iv))	2.15
	0.977	7^+	$3\ 7/2$	3.39(iii) (4.00 (iv))	3.02

(i) The $^{42}\text{Ca}(\text{d},\text{p})^{43}\text{Ca}$ reaction ($E_{\text{d}}=7$ MeV) from Brown *et al* 1974

(ii) The $^{44}\text{Ca}(\text{d},\text{p})^{45}\text{Ca}$ reaction ($E_{\text{d}}=7$ MeV) from Brown *et al* 1974

(iii) The $^{45}\text{Sc}(\text{d},\text{p})^{46}\text{Sc}$ reaction ($E_{\text{d}}=12$ MeV) from ref. Roy *et al* 1992

(iv) The $^{45}\text{Sc}(\text{d},\text{p})^{46}\text{Sc}$ reaction ($E_{\text{d}}=7$ MeV) from Rappaport *et al* 1966

Table 4. Comparison of the spectroscopic factors for reactions involving ground states of Ca nuclei calculated:

(i) by assuming a pure $(0f_{7/2})^m$ configuration for neutrons above the ^{40}Ca core (second column),

(ii) in the full $(1p0f)^m$ shell-model space (third column).

Reaction	spectroscopic factor	
	pure $(0f_{7/2})^m$ model	full $(1p0f)^m$ model
$^{40}\text{Ca}(\text{d},\text{p})^{41}\text{Ca}$	1	1
$^{42}\text{Ca}(\text{d},\text{p})^{43}\text{Ca}$	0.75	0.75
$^{44}\text{Ca}(\text{d},\text{p})^{45}\text{Ca}$	0.5	0.507
$^{46}\text{Ca}(\text{d},\text{p})^{47}\text{Ca}$	0.25	1.036
$^{44}\text{Ca}(\text{p},\text{d})^{43}\text{Ca}$	4	3.644
$^{42}\text{Ca}(\text{p},\text{d})^{41}\text{Ca}$	2	1.812

Figure captions

Figure 1. The percentage distribution of the total strength among states of ^{42}Ca produced by the neutron pick-up from the ^{43}Ca target. States of ^{42}Ca are specified by the spin J , parity Π , isospin T and calculated excitation energy E_{calc} .

Figure 2. The percentage distribution of the total strength among states of ^{44}Sc produced by the neutron pick-up from the ^{45}Sc target. Theoretical results (first columns) are compared with the experimental data (second ones) obtained from the $^{45}\text{Sc}(\text{d},\text{t})^{44}\text{Sc}$ reaction (Ohnuma and Sourkes 1971).

Figure 3. The percentage distribution of the total strength among states of ^{47}Ca produced by the neutron stripping on the ^{46}Ca target.

Figure 4. The measured transition strength $((2J_f + 1)/(2J_i + 1)) S$ for states of ^{46}Sc produced by the neutron stripping on the ^{45}Sc target compared to the theory.

- (a): theoretical predictions from McCullen *et al* 1964,
- (b) (d,p) reaction from Roy *et al* 1992,
- (c) (d,p) reaction from Rappaport *et al* 1966,
- (d) (t,d) reaction from Bing *et al* 1976,
- (e) theory of the present work.